Intensive Longitudinal Data: A Multilevel Modeling Perspective

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APA Science Training Sessions:
The Collection and Analysis of Intensive Longitudinal Data

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Intensive Longitudinal Data

Part 2: A Multilevel Modeling Perspective

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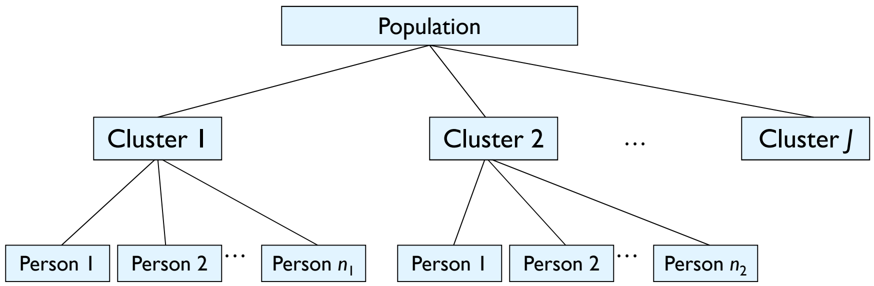
Objectives

- Conceptualize ILD as a nested data structure
- Present basic MLM for decomposing within v. between person variance
- Differentiate within- and between-person effects of predictors in MLM
- Use MLMs to capture time trends that vary between people

2.2

Nested Data: Hierarchical Case

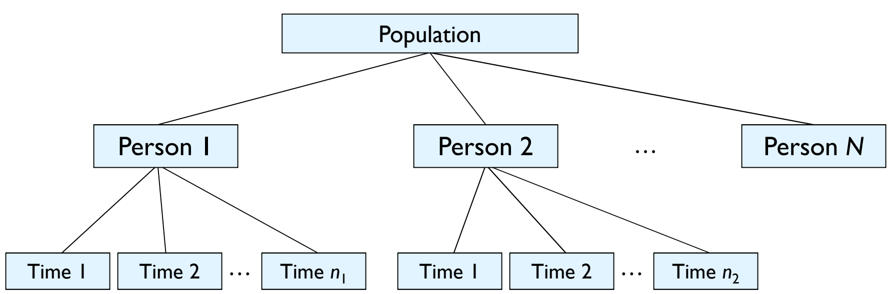
- MLM was originally developed for hierarchically nested data consisting of people nested within clusters
  - Classic Case: Students nested within schools



2.3

Longitudinally Nested Data

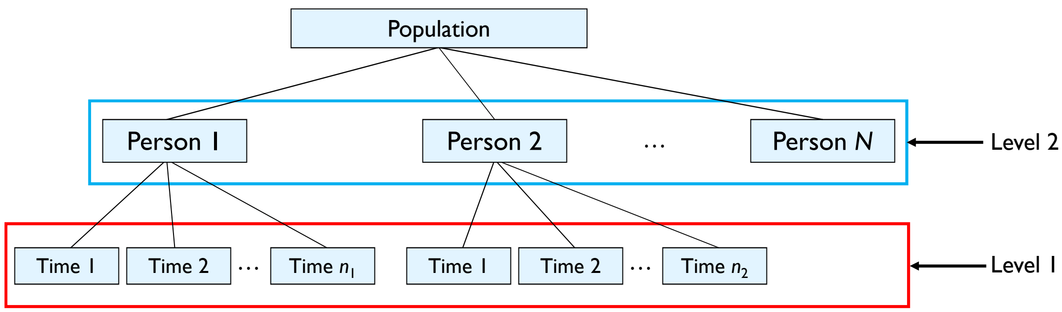
- Longitudinal data can similarly be regarded as hierarchically nested
  - Classic Case: Long-Term Longitudinal Data / Panel Data
  - Also ILD, just fewer people and more time points



2.4

Levels of Sampling

- In MLMs, we conceptualize data as arising from two levels of sampling
  - Hierarchical: we sample schools, then students within schools
  - Longitudinal: we sample individuals, then repeated measures within individuals
- Higher level is called "Level 2" and lower level is "Level 1"



2.5

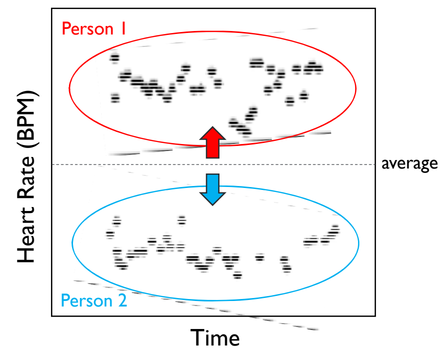
Sources of Variability

- When we consider nested data, we need to conceptualize sources of variance at each level of sampling
  - In traditional general linear model (GLM; e.g., ANOVA or regression) there is only one source of variability captured in the residual variance (or MSE)
  - But in nested data there are *multiple* sources of variability
- Example: Ambulatory heart rate (HR) readings on a set of individuals
  - **Between-person variability at Level 2**: Some people have higher average HRs than other people
  - **Within-person variability at Level 1**: At some moments in time, a person's HR is elevated relative to their own average compared to other moments in time

2.6

Dependence in ILD

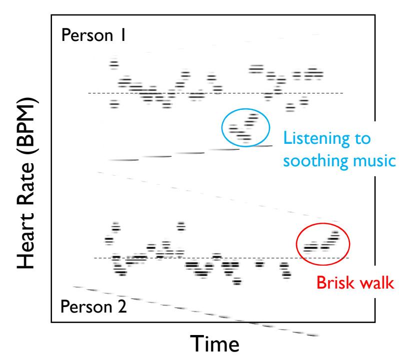
- At Level 2, between-person differences lead observations to be higher (or lower) for some people than others
  - HR observations on people with poor cardiovascular health will often be higher than the across-persons average
  - HR observations on people with good CV health will often be lower than average
- Observations are thus positively correlated within person
  - If your overall level is higher, your repeated measures tend to be higher, and vice versa



2.7

Dependence in ILD

- At Level 1, serial correlation leads consecutive observations for same person to look similar
  - Person 1 listens to soothing music at midday, shows dip in HR relative to their baseline
  - Person 2 takes a brisk walk at the end of the day, shows increased HR relative to baseline
- Observations close together in time tend to be highly correlated within person
  - Correlation typically "decays" with increasing spacing in time (e.g. correlation of two morning measures vs. a morning and afternoon measure)



2.8

Modeling ILD

- Need to address & disambiguate sources of variability across levels, both to properly represent sample data and to test theoretical hypotheses
  - Systematic variability (associated with predictors in model)
  - Random or residual variability (unexplained by predictors in model)
- In doing so, represent processes that give rise to dependence in the data
  - Dependence due to between-person differences
  - Dependence due to within-person serial correlation
- Failure to attend to these issues can lead to serious misinterpretations of effects and inaccurate inferences, undermining internal validity of substantive interpretations
  - Misattribution of causal inferences

2.9

Random-Effects ANOVA Model

- We start with a *random-effects ANOVA* model (a.k.a. *empty* or *null* model) to show how MLM decomposes variance and captures dependence
- *Level 1 equation* expresses intra-individual (or within-person) variability:

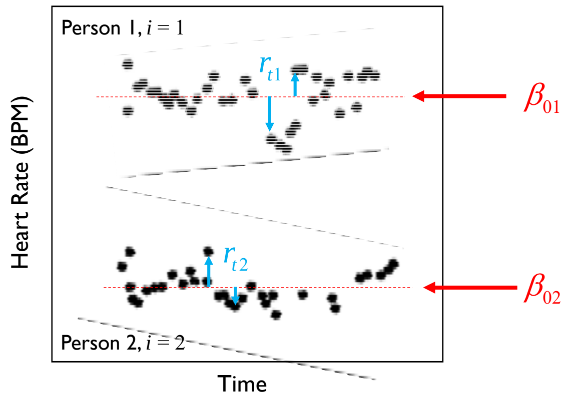
$$y_{ti} = \beta_{0i} + r_{ti}$$

- β_{0i} is an intercept for person i that captures their over-time mean of y
  - In absence of predictors, intercept = mean
- r_{ti} captures how the value of y for person i at time t varies from this person mean
  - e.g., a positive value for r_{ti} indicates score on y at time t was above person's overall average, and vice versa

2.10

Visualizing Level 1 Equation

$$y_{ti} = \beta_{0i} + r_{ti}$$



Notice that r_{ti} captures intra-individual or within-person variability

Often assume equal variance in r_{ti} across persons, but also possible some individuals have "tighter" (or less variable) distributions around their overall mean while others have "looser" (or more variable) distributions

2.11

Random-Effects ANOVA Model

- *Level 2 equations* express inter-individual (or between-person) variation
- Each coefficient at Level 1 (e.g., β_{0i}) gets a Level 2 equation:

Level 1: $y_{ti} = \boxed{\beta_{0i}} + r_{ti}$

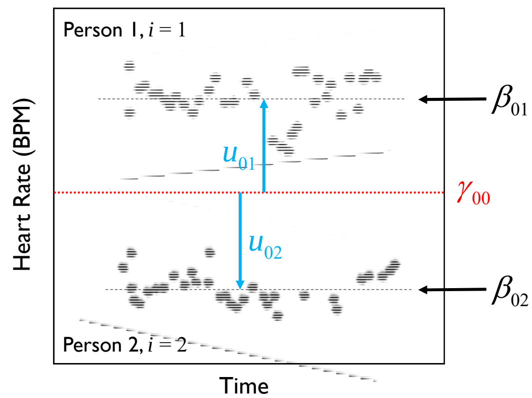
Level 2: $\boxed{\beta_{0i}} = \gamma_{00} + u_{0i}$

- Here γ_{00} is an intercept that represents the across-persons mean of β_{0i}
  - In absence of predictors at either level, an overall average for y
- u_{0i} captures how the value of β_{0i} (mean for person i) varies from γ_{00} (the across-persons average)
  - e.g., a positive value for u_{0i} indicates person's average is higher than the across-persons average, and vice versa

2.12

Visualizing Level 2 Equation

$$\beta_{0i} = \gamma_{00} + u_{0i}$$



Notice that u_{0i} captures inter-individual or between-person variability

The inclusion of this term accounts for one source of dependence in the data, e.g., that person 1 has a higher average than person 2

2.13

Reduced-Form / Mixed Model Equation

- Substituting Level 2 into Level 1, we obtain the *reduced form* model:

Level 1: $y_{ti} = \beta_{0i} + r_{ti}$

Level 2: $\beta_{0i} = \gamma_{00} + u_{0i}$

Reduced Form: $y_{ti} = \gamma_{00} + u_{0i} + r_{ti}$

- *Fixed effects* (denoted by γ) take on a constant value for all observations
  - Similar to usual regression coefficients that represents a relation for <u>all</u> subjects
- Residuals & *random effects* (r at Level 1 & u at Level 2) take on a <u>distribution</u> of potential values
  - Similar to usual error term in regression but now at each level of sampling and thus represents multiple "sources" of variability in the model

2.14

Variance Components

- We posit a distribution for each residual / random effect:

$$\text{Level 1:} \quad y_{ti} = \beta_{0i} + r_{ti} \qquad r_{ti} \sim N\left(0, \sigma^2\right)$$

$$\text{Level 2:} \quad \beta_{0i} = \gamma_{00} + u_{0i} \qquad u_i \sim N\left(0, \tau_{00}\right)$$

$$\text{Reduced Form:} \quad y_{ti} = \gamma_{00} + u_{0i} + r_{ti}$$

- The random-effects ANOVA thus consists of three parameters:
  - γ_{00} = average of y across persons and over time
  - σ^2 = within-person variance in y
  - τ_{00} = between-person variance in y

  (Variance Components: σ^2 and τ_{00})
- In traditional GLM we include σ^2 but assume τ_{00} does not exist
  - *la la la la la I can't hear you……*

2.15

The Intra-Class Correlation

- Can compute degree of dependence in data with the *intra-class correlation*:

$$\text{ICC} = \frac{\tau_{00}}{\tau_{00} + \sigma^2}$$

- Simple ratio of between-person variance divided by total variance
- ICC has two equivalent and equally valid interpretations:
  - Proportion of total variance that is due to between-person differences
  - Within-person correlation of observations over time
- For example, if an ICC were equal to .25:
  - Of the total observed variability, 25% is due to between-person differences
  - On average, repeated measures are correlated .25 within individuals

2.16

Measures of Heart Rate Throughout The Day

- Sample obtained from Laura Richman at Duke University:
  - 61 individuals between age 18 and 53, 49% African-American, 34% male
  - Heart Rate (HR) and blood pressure measured over 24 hours: 3 per hour while awake, 2 per hour while asleep (range of 6-60 assessments)
- Initially focus on daytime HR observations
  - These are the same data we presented in Day 1 in tabular and graphical form

Richman, L.S., Pek, J., Pascoe, E., & Bauer, D.J. (2010). The effects of perceived discrimination on ambulatory blood pressure and affective responses to interpersonal stress modeled over 24 hours. *Health Psychology, 29*, 403-411. DOI: 10.1037/a0019045

2.17

Example: Daytime Heart Rates

- We want to decompose variance into within- and between-person components and estimate ICC
- Fit Random Effects ANOVA:
  - $\hat{\gamma}_{00} = 79.22$ is overall average HR
  - $\hat{\tau}_{00} = 79.78$ is between-persons variance in HR
  - $\hat{\sigma}^2 = 147.89$ is within-persons variance in HR

$$\text{ICC} = \frac{79.78}{79.78 + 147.89} = .35$$

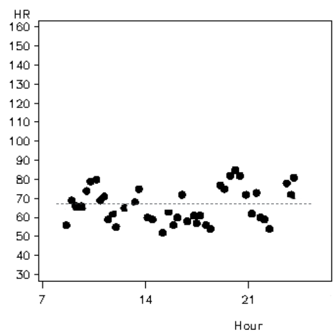
35% of HR variance due to between-person differences

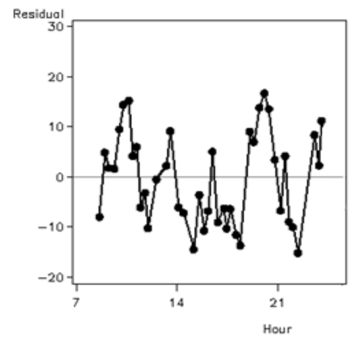
Correlation of repeated assessments of HR within person is .35

2.18

What About Serial Correlation?

- Nesting of repeated measures within individual one source of dependence
- But we have not yet modeled the *second* source of dependence common to ILD: namely, *serial correlation*
- The apparent cycling seen for the first person ($i = 1$) is a tell-tale sign:





2.19

Introducing Serial Correlation

- In MLM, customary to assume that Level-1 residuals are *independent*, a condition often violated with ILD
  - however, there are a variety of residual correlation structures we can consider
- With unequal time intervals (common in ILD applications), one option is a continuous-time autoregressive residual structure:

$$CORR(r_{ti}, r_{t'i}) = \rho^{|time_{t'i} - time_{ti}|}$$

- Introduces one new parameter, ρ, to capture autoregression
  - The necessity of ρ is a testable hypothesis
- With $\rho < 1$, correlation between observations decays as a power of the distance in time, so correlations get smaller and smaller with time

2.20

Example: Daytime Heart Rates

- We expand prior MLM to include continuous-time AR residual correlation
- New results:
  - $\hat{\gamma}_{00} = 79.09$ is overall average HR
  - $\hat{\tau}_{00} = 77.87$ is between-persons variance in HR (smaller than before)
  - $\hat{\sigma}^2 = 154.76$ is within-persons variance in HR (larger than before)
  - $\hat{\rho} = .066$ captures serial correlation of within-person residuals

15 minutes apart: $.066^{.25} = .507$

30 minutes apart: $.066^{.50} = .257$

60 minutes apart: $.066^{1.0} = .066$

- Inclusion of AR parameter significantly improves our ability to properly represent characteristics of sample data

2.21

Predictors at Level 1

- We can now introduce predictors at each level of the model to capture systematic sources of this within- and between-person variability
- Predictors at Level 1 are called time-varying covariates (or TVCs)
  - HR example: posture, perceived stress, social engagement with others, etc.
- Level 1 predictors contain variability both within- and between-persons
  - Dan alternates between standing and sitting (within-person variability); throughout the day, Patrick sits more than Dan does (between-person variability)
- To capture within-person effects, we person-mean center Level 1 predictors
  - Literally subtract each person's average value from their scores
- For between-person effects of TVC, can add person means as Level 2 predictor
  - See Curran & Bauer (2011, *Annual Review of Psychology*) for more details than you want

2.22

Inclusion of Level 1 Predictor

- Augment our Level 1 equation with new predictor(s)
- Each additional coefficient at Level 1 results in a new Level 2 equation:

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i} x_{1ti} + r_{ti}$ ← Value of TVC linked to time *t* for individual *i*

Level 2: $\beta_{0i} = \gamma_{00} + u_{0i}$ ← Random intercept

$\beta_{1i} = \gamma_{10} + u_{1i}$ ← Random slope (optionally included)

- Need to decide whether to include random slope: implies within-person effect varies in magnitude across people
  - Stress impacts HR differently for different people
  - Can make necessity of random slope a testable hypothesis

2.23

Adding Predictors at Level 2

- Predictors at Level 2 are called time-invariant covariates (TICs) and predict between-person differences in outcome
- TICs are person-level "time free" individual difference variables
  - For HR: age, biological sex, self-identified race, BMI, perceived discrimination
- Can expand Level 2 Equations to include one or more TICs:

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01} w_{1i} + \gamma_{02} w_{2i} + u_{0i}$

- As noted earlier, could also include aggregated variables reflecting between-person variability in Level 1 predictors (person means of TVCs)
  - For HR example: proportion of time points sitting, average report of stress

2.24

Cross-Level Interactions

- Can also include Level 2 predictors of slopes:
  - Is the time-linked effect of stress on HR smaller in magnitude for older people?

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}x_{1ti} + r_{ti}$

Level 2: $\beta_{0i} = \gamma_{00} + \boxed{\gamma_{01}w_{1i}} + u_{0i}$ ← TIC prediction of intercept

$\beta_{1i} = \gamma_{10} + \boxed{\gamma_{11}w_{1i}} + u_{1i}$ ← TIC prediction of slope

Reduced Form: $y_{ti} = (\gamma_{00} + \gamma_{01}w_{1i} + u_{0i}) + (\gamma_{10} + \boxed{\gamma_{11}w_{1i}} + u_{1i})x_{1ti} + r_{ti}$

$= (\gamma_{00} + \gamma_{01}w_{1i} + \gamma_{10}x_{1ti} + \boxed{\gamma_{11}w_{1i}x_{1ti}}) + (u_{0i} + u_{1i}x_{1ti} + r_{ti})$ ← Cross-Level Interaction

  - see Bauer & Curran (2005, *Multivariate Behavioral Research*) for more details than you want

2.25

HR Example

- We observed variance within-person (time-specific variability in HR around overall person mean) and between-person (some people have higher average HR and others lower)
- Now want to predict these two sources of variability
  - Within- and between-person effects of *sitting*
  - Between-person effects of *age*, *sex*, and *perceived discrimination*
  - Possible cross-level interaction of *age* and *sitting*

Level 1: $y_{ti} = \beta_{0i} + \beta_{1i}Sit_{ti} + r_{ti}$

Level 2: $\beta_{0i} = \gamma_{00} + \gamma_{01}SitMean_i + \gamma_{02}Male_i + \gamma_{03}Age_i + \gamma_{04}PD_i + u_{0i}$

$\beta_{1i} = \gamma_{10} + \gamma_{11}Age_i + u_{1i}$

2.26

Results Summary

- Little difference in within vs. between effects of sitting: -11.22 vs. -13.89
  - When people sit, we observe lower HRs, and these effects are comparable in magnitude within a given person and across persons
  - Kind of boring, but other time-varying predictors might show more interesting differences, such as perceived stress.
- Average HR did not differ by sex
- Average HR significantly increased with age
- Higher perceived discrimination predicted higher average HR
- Trend for smaller decrease in HR when sitting at older ages, but non-significant

2.27

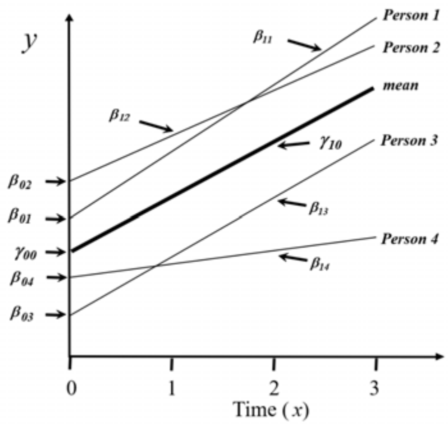
Change Over Time

- So far, have focused on unpacking within- v. between-person variability in outcomes and predicting that with Level 1 and Level 2 predictors
- But we have assumed people are fluctuating around average values that are stable over time
- What if people are changing systematically over time?
  - That is, increasing or decreasing with the passage of time
- Examining *time trends* is another common goal in ILD analyses
  - Seek to model inter-individual (between-person) differences in intra-individual (within-person) change
- MLM exceptionally well suited to incorporate these effects

2.28

A Linear Growth Model

- Key to modeling trajectories in ILD is to include *time* as a level 1 predictor
  - identical to prior model, but now individual trajectories can "tilt" up or down



$$y_{ti} = \beta_{0i} + \beta_{1i} time_{ti} + r_{ti}$$

$$\beta_{0i} = \gamma_{00} + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$

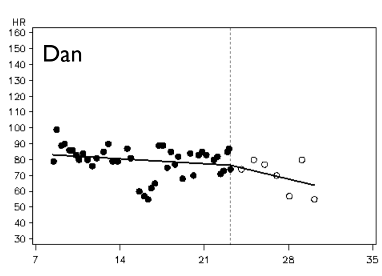
Random effects capture between person differences in change

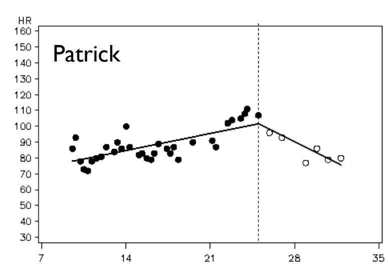
Fixed effects capture average trajectory

2.29

Piecewise Models for Transitions

- Often interest is in studying change over distinct phases
  - Pre-treatment versus post-treatment
  - Pre-event versus post-event
- HR during *awake* versus *asleep* where transition to sleep varies by person:





2.30

Piecewise Linear Model for Heart Rate

- To capture slopes in different phases, decompose time axis into multiple variables
  - e.g., one time variable when *awake* and another time variable when *asleep*

Level 1:

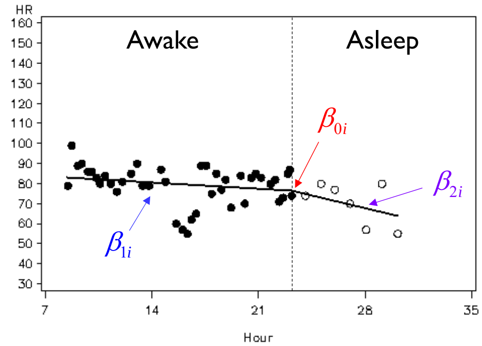
$$y_{ti} = \beta_{0i} + \beta_{1i} timewake_{ti} + \beta_{2i} timesleep_{ti} + r_{ti}$$

Level 2:

$$\beta_{0i} = \gamma_{00} + u_{0i}$$
$$\beta_{1i} = \gamma_{10} + u_{1i}$$
$$\beta_{2i} = \gamma_{20} + u_{2i}$$

Reduced Form:

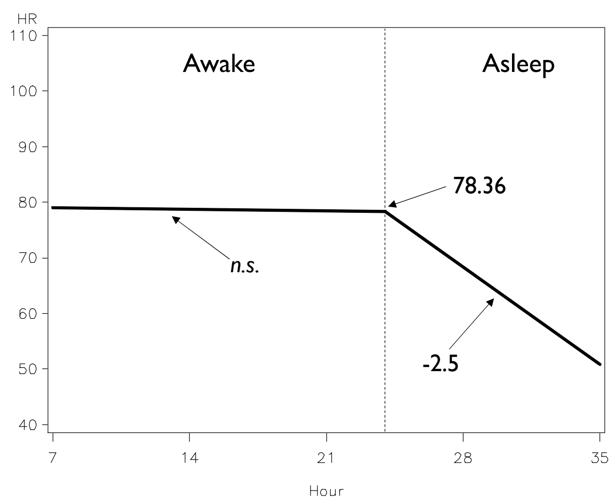
$$HR_{ti} = (\gamma_{00} + \gamma_{10} timewake_{ti} + \gamma_{20} timesleep_{ti}) + (u_{0i} + u_{1i} timewake_{ti} + u_{2i} timesleep_{ti}) + r_{ti}$$



2.31

Average Trajectory: Fixed Effects

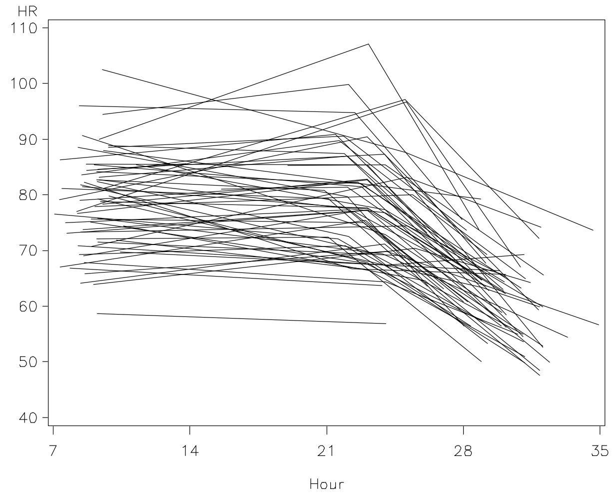
- At last assessment before going to sleep, expected HR = 78.36
- During the waking period, HR shows no significant time trend
- During the sleeping period, HR dips at a rate of 2.5 BPM/hour



2.32

Individual Trajectories (Random Effects)

- Significant variance in all three trajectory components
- Lots of variation in level at going to bed
- Lots of variation in slopes during both day and night
- Variation in "dipping" at night of particular interest



2.33

Predictors of Change

- Could next include predictors of rates-of-change at Level 2
  - Between-person differences in within-person change
- With heart rate data, saw earlier that perceived discrimination predicted higher daytime HR
- Does perceived discrimination predict less night-time dipping?
  - Include *perceived discrimination* in slope equation for *timesleep*
  - Results in cross-level interaction of *PD* with *timesleep* in reduced form
- We do not show results here, but model can be expanded in variety of ways to test specific theoretically-derived hypotheses about HR throughout the day

2.34

Summary

- MLM treats ILD as nested with repeated measures nested within person
- Need to account for dependence due to between-person differences *and* within-person serial correlation
- Need to differentiate prediction of within- versus between-person variability in outcomes
- Areas of particular focus in analysis of ILD are
  - Within-person effects (processes as they operate within persons)
  - Between-person differences in within-person effects (how do processes differ across persons)
  - Between-person differences in trajectories (often before/after some event)
- MLMs provide a flexible framework for conducting these analyses

2.35

Where We Go Next: The Dynamic SEM

- Today we discussed ILD from the perspective of the multilevel model
- Dynamic structural equation modeling (or DSEM) offers another modeling perspective for intensive longitudinal data
- Like the GLM, MLM focuses on *one outcome* and assumes measures are *error free*
  - Single outcome limits hypotheses we can test
  - Unreliability can bias parameter estimates, potential under-estimation of effects
- DSEM offers a multivariate framework for analyzing ILD, allowing for multiple outcomes with potentially reciprocal or cascading effects and/or latent factors to account for measurement error
- Join us on Day 3 to learn more!

2.36

A Semi-Random Sampling of Resources

Bauer, D. J. & Curran, P. J. (2005). Probing interactions in fixed and multilevel regression: inferential and graphical techniques. *Multivariate Behavioral Research, 40*, 373-400.

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Curran, P. J. & Bauer, D. J. (2011). The disaggregation of within-person and between-person effects in longitudinal models of change. *Annual Review of Psychology, 62*, 583-619.

Hedeker, D., & Gibbons, R. D. (2006). *Longitudinal data analysis*. Wiley-Interscience.

Hedeker, D., Mermelstein, R. J., & Demirtas, H. (2008). An application of a mixed-effects location scale model for analysis of ecological momentary assessment (EMA) data. *Biometrics, 64*, 627-634.

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Hox, J. J., Moerbeek, M., & Van de Schoot, R. (2017). *Multilevel analysis: Techniques and applications*. Routledge.

McCoach, D. B. (2010). Hierarchical linear modeling. *The reviewer's guide to quantitative methods in the social sciences*, 123-140.

Raudenbush, S. W. (2001). Comparing personal trajectories and drawing causal inferences from longitudinal data. *Annual Review of Psychology, 52*, 501.

Raudenbush, S. W., & Bryk, A. S. (2002). *Hierarchical linear models: Applications and data analysis methods* (Vol. 1). Sage.

Snijders, T. A., & Bosker, R. J. (2011). *Multilevel analysis: An introduction to basic and advanced multilevel modeling*. Sage.

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A Semi-Random Sampling of Software Guides

- SAS:
  - Singer, J.D. (1998) Using SAS PROC MIXED to fit multilevel models, hierarchical models, and individual growth models. *Journal of Educational and Behavioral Statistics, 23,* 323-355.
- SPSS:
  - Peugh, J.L. & Enders, C.K. (2005). Using the SPSS MIXED procedure to fit cross-sectional and longitudinal multilevel models. *Educational and Psychological Measurement, 65*, 717-741
- Stata:
  - Rabe-Hesketh, S. & Skrondal, A. (2022). *Multilevel and longitudinal modeling using Stata, Volumes I and II* (4th Ed.). Stata Press.
- R:
  - Finch, W.H., Bolin, J.E. & Kelly, K. (2014). *Multilevel modeling using R.* CRC Press / Taylor & Francis
  - Shaw, M. & Flake, J.K. (2022). *Introduction to multilevel modeling*. learn-mlms.com

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