Introduction to Latent Curve Modeling

Daniel J. Bauer & Patrick J. Curran
University of North Carolina at Chapel Hill

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Section 1

Defining the Latent Curve Model

Cohort-by-Age Design Structure

- A cohort-by-age table is helpful to organize different designs
  - **rows** are *birth cohorts* (or *birth years*) and **columns** are *chronological ages*
  - **cells** are the year in which a child from a given cohort was a given age
  - for example, a child in the 1990 cohort was 16 years of age in 2006

	Chronological Age						
Cohort	10	12	14	16	18	20	22
1986	1996	1998	2000	2002	2004	2006	2008
1988	1998	2000	2002	2004	2006	2008	2010
1990	2000	2002	2004	2006	2008	2010	2012
1992	2002	2004	2006	2008	2010	2012	2014
1994	2004	2006	2008	2010	2012	2014	2016
1996	2006	2008	2010	2012	2014	2016	2018

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Cross-Sectional Design

- A more common design is a single assessment collected in 2008, at which multiple ages are assessed as a ***cross-section*** of ages 12 to 22
- This is strictly a *between-person* design as each individual provides just one assessment at their given age in 2008

	Chronological Age						
Cohort	10	12	14	16	18	20	22
1986	1996	1998	2000	2002	2004	2006	$\boxed{2008}$
1988	1998	2000	2002	2004	2006	$\boxed{2008}$	2010
1990	2000	2002	2004	2006	$\boxed{2008}$	2010	2012
1992	2002	2004	2006	$\boxed{2008}$	2010	2012	2014
1994	2004	2006	$\boxed{2008}$	2010	2012	2014	2016
1996	2006	$\boxed{2008}$	2010	2012	2014	2016	2018

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Multiple Cohort Longitudinal Design

- Instead of following a single cohort over time, could simultaneously follow multiple cohorts
- Spans seven years of age, but no child assessed more than 3 times
  - thus sometimes called an *accelerated longitudinal design*
  - allows for specific tests of **cohort effects**

	Chronological Age						
Cohort	10	12	14	16	18	20	22
1986	1996	1998	2000	2002	2004	2006	2008
1988	1998	2000	2002	2004	2006	2008	2010
1990	2000	2002	2004	2006	2008	2010	2012
1992	2002	2004	2006	2008	2010	2012	2014
1994	2004	2006	2008	2010	2012	2014	2016
1996	2006	2008	2010	2012	2014	2016	2018

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Longitudinal Data & Intra-Individual Change

- Goals of longitudinal data analysis:
  1. establish temporal precedence
  2. characterize the pattern of change ***within*** each individual
  3. examine the individual-change patterns ***across*** individuals
- This allows for the explicit examination of *between*-person differences in *within*-person change
  - ***inter-individual differences*** in ***intra-individual change***
- Separation of within- & between-person change is key advantage of longitudinal data and will be focus here
  - indeed, this might be **most** important advantage of longitudinal data

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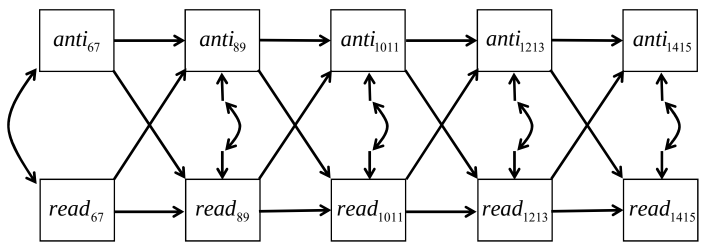
Traditional Autoregressive Crosslagged Model

- The ARCL model is a traditional regression-based model that predicts later behavior from earlier behavior & is based on "residualized" change



- But there are many limitations of this approach, particularly if theory posits individual-specific continuous trajectories of change

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The Latent Growth Curve

- Many substantive theories posit smooth trajectories of change over continuous time
- Will build model for data that estimates change over time <u>within</u> each individual and then compare change <u>across</u> individuals
  - e.g., estimate *inter-individual* variability in *intra-individual* change
- This is core concept behind a growth curve
  - also sometimes called latent trajectories, latent curves, growth trajectories, or time paths

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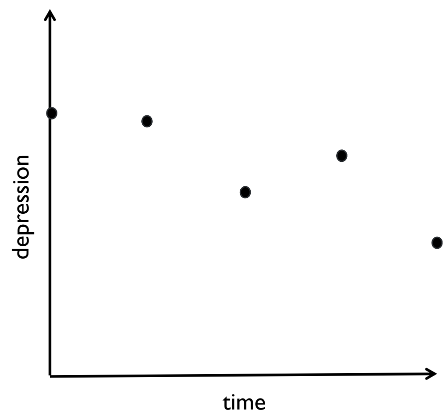
Repeated Measures for One Person

- Consider hypothetical case where we have five repeated measures assessing depression in a single adolescent



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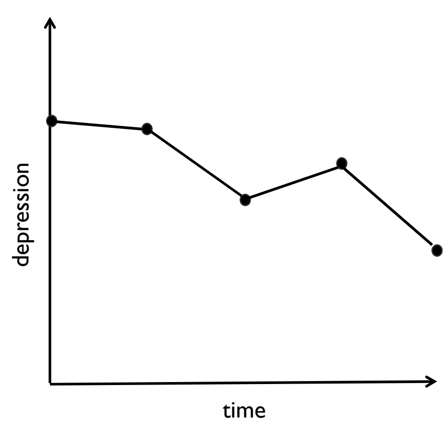
Repeated Measures for One Person

- Could connect observations to see time-adjacent changes



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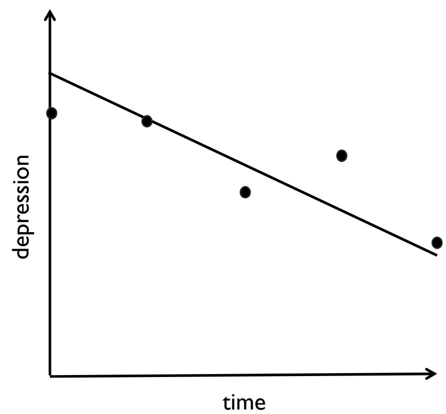
A Growth Curve for One Person

- Could instead "smooth over" repeated measures and estimate a line of best fit for this individual



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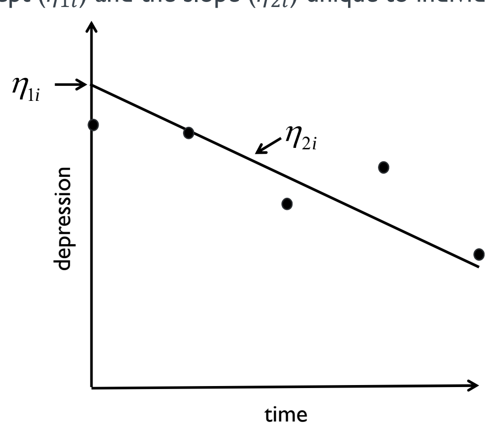
A Growth Curve for One Person

- Can summarize line by two pieces of information
  - the intercept (η_{1i}) and the slope (η_{2i}) unique to individual i



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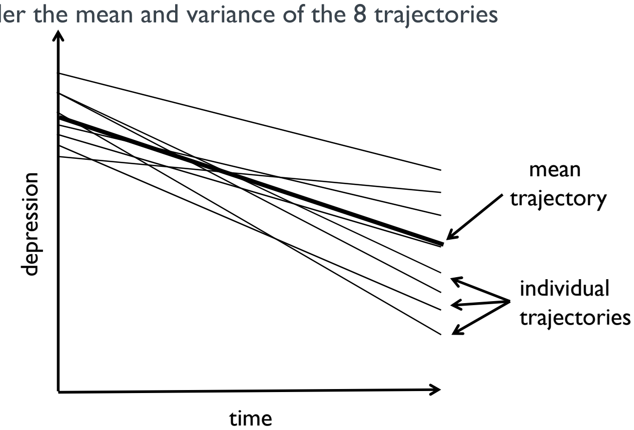
Growth Curves for Multiple Persons

- Rarely interested in one individual, but in a sample of individuals
  - can extend to 8 trajectories (but would use 100 or more in practice)
  - can also consider the mean and variance of the 8 trajectories



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The Latent Growth Curve

- Characteristics of the latent trajectories captured in two ways
- Trajectory means
  - the average value of the parameters that define the growth trajectory pooling over all individuals in the sample
- Trajectory variances
  - the variability of individual cases around the mean trajectory parameters
  - larger variances reflect greater variability in growth
- Can consider various restrictions on these parameters to model different patterns of growth over time

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Initial Motivating Questions

- What is the mean course of change over time?
  - linear, quadratic, exponential, no change, etc.
- Are there individual differences in the course of change?
  - variability in starting point and rate of change
- Are there time-invariant or time-varying predictors of change?
  - person-level characteristics like gender, ethnicity diagnostic status
  - time-specific characteristics like onset of drinking, arrest, marriage
- Do two constructs travel together through time?
  - trajectories of alcohol use linked with trajectories of depression
- Do trajectories vary over observed or latent groups?

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Capturing Growth as a Latent Factor

- Theory posits existence of unobserved continuous trajectory
- Cannot directly observe, but can infer existence based on set of repeated measures
- Growth curve thus fits naturally into latent variable model
  - e.g., depression, self esteem, worker productivity, etc.
- Can draw on strengths of confirmatory factor analysis (CFA) to define latent curve model
- The LCM is fundamentally a highly restricted CFA model

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Confirmatory Factor Analysis

- Primarily theory-driven: test model that specifies the number and nature of the latent factors behind set of observed measures
  - e.g., latent depression and anxiety underlie set of 20 symptom items
- Model identified through restrictions on parameters
- Number of latent factors determined by theory
- Factor pattern matrix is restricted by analyst to reflect theory
  - e.g., some loadings freely estimated, others fixed to zero
- Attention paid to global and local fit of model to data

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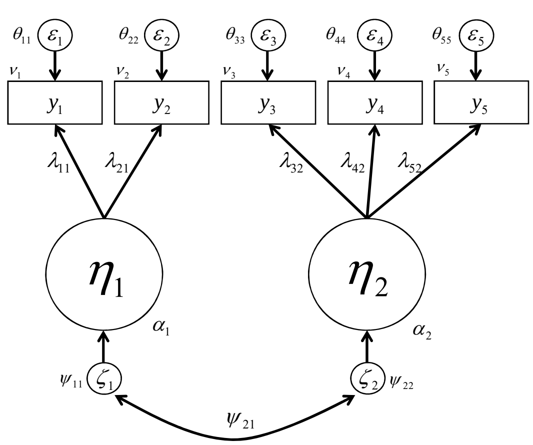
The Parameters of the General CFA



- y observed item
- ε item residual
- θ residual variance
- ν item intercept
- λ factor loading
- η factor score
- α factor mean
- ζ factor disturbance
- ψ factor co/variance

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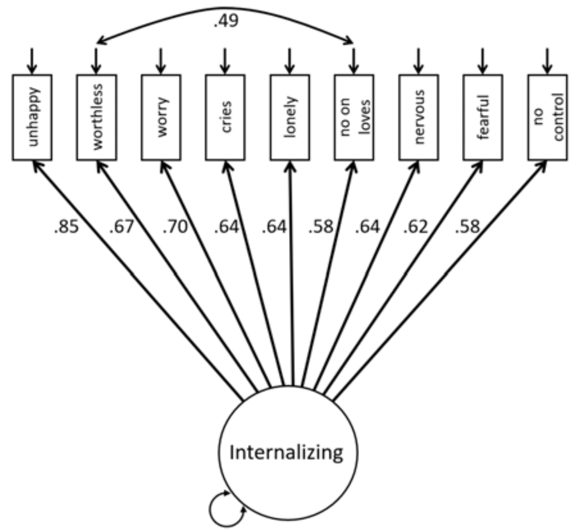
Example of a One-Factor CFA

- 9-item, one-factor CFA of internalizing symptomatology for a sample of 394 adolescents aged 12 to 18
- Model fit is reasonably good
  - $\chi^2(26) = 53.23, p = .0013$
  - $RMSEA = .052, CFI = .94, TLI = .96$
- Standardized loadings are presented in the path diagram
- We thus infer what we believe to exist but did not directly observe (the latent factor) from what we did observe (the items)



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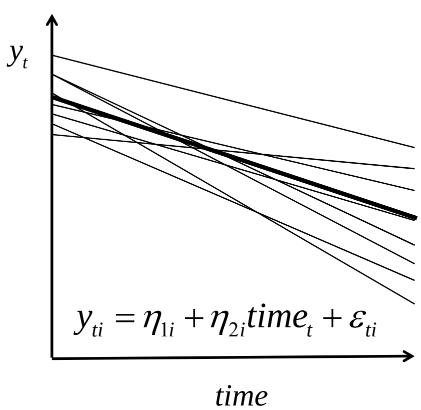
Returning to the Latent Curve

- We believe individual trajectories exist for each case, but these were not observed directly
  - want to infer the latent curves based on the data that were observed



- We can think of estimating individual trajectories through the regression of the repeated measures on the *time*
- It is good-old "rise-over-run"; that is, there is some expected change in the outcome per unit-change of time
- We do this through the CFA

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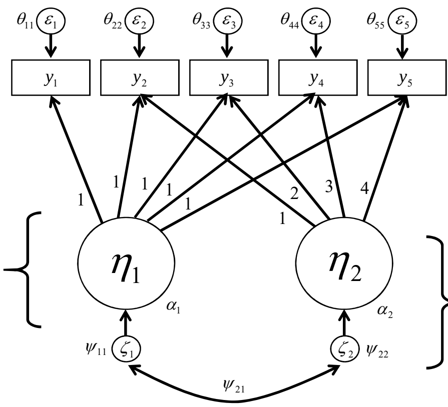
A Linear LCM as a CFA



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i$$

Numerical values of *time* entered in factor loading matrix

Intercept and Slope Factors

$$\boldsymbol{\eta}_i = \boldsymbol{\alpha} + \boldsymbol{\zeta}_i$$

- Notice time embedded in factor loading matrix
  - Need to think about coding of time and how that influences interpretation

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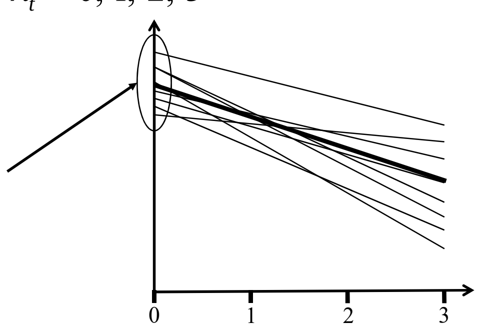
Coding Time: Zero Point

- Most common to set numerical value of time to equal zero at the first assessment period:

$$\lambda_t = 0,\ 1,\ 2,\ 3$$

Mean and variance of intercept is scaled for <u>initial</u> time period because when time is zero the slope factor does not contribute



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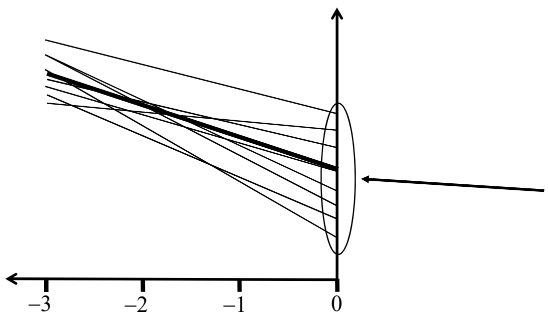
Coding Time: Zero Point

- Can instead code time such that the intercept defines the *last* assessment

$$\lambda_t = -3, -2, -1, 0$$



Mean and variance of intercept scaled to reflect the <u>last</u> time period; might be useful if conducting a treatment evaluation

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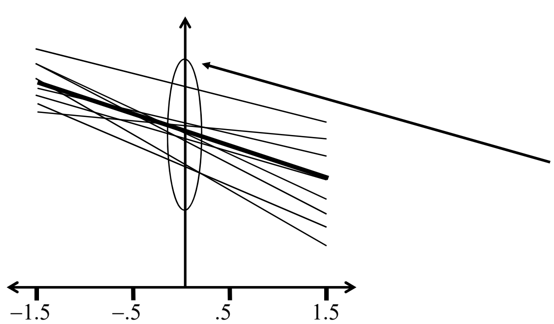
Coding Time: Zero Point

- Can even code time so the intercept defines the *middle* assessment

$$\lambda_t = -1.5, -.5, .5, 1.5$$



Mean and variance of intercept scaled to reflect the <u>middle</u> time period; for case of even number of assessments, middle may not have been directly observed but is implied by model

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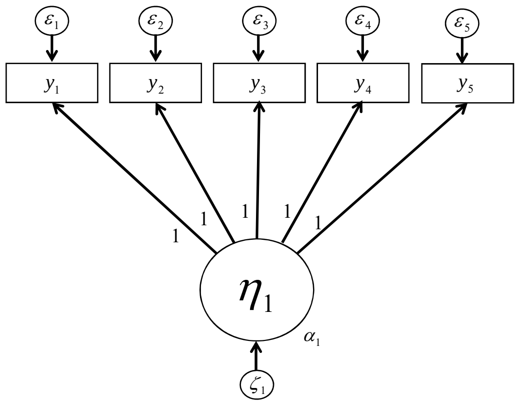
Intercept-only LCM

- Implies outcome does not change as function of time
  - all individual trajectories horizontal but at different levels



$$\mathbf{y}_i = \boldsymbol{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i$$

$$\begin{bmatrix} y_{1i} \\ y_{2i} \\ y_{3i} \\ y_{4i} \\ y_{5i} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} \eta_{1i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \\ \varepsilon_{5i} \end{bmatrix}$$

$$\boldsymbol{\eta}_i = \boldsymbol{\alpha} + \boldsymbol{\zeta}_i$$

$$\begin{bmatrix} \eta_{1i} \end{bmatrix} = \begin{bmatrix} \alpha_1 + \zeta_{1i} \end{bmatrix}$$

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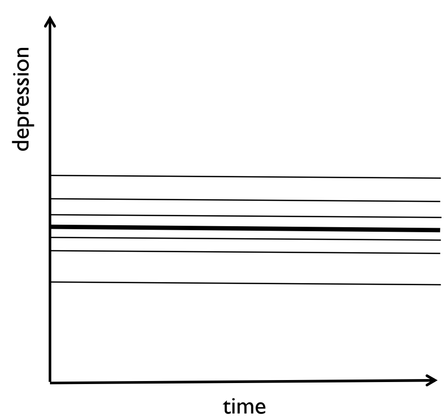
Intercept-only LCM: Trajectories

- Intercept-only LCM implies between-person variability in overall level of outcome, but outcome does not change with time



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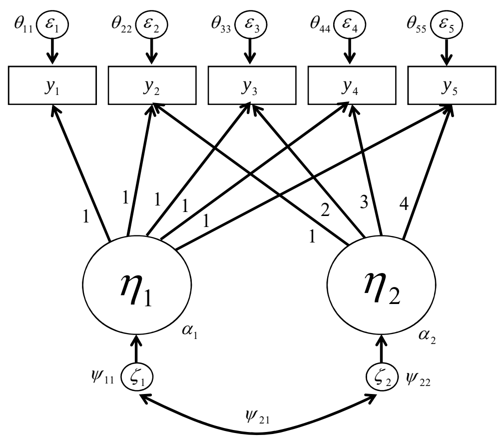
Linear LCM

- Can add a second correlated factor to capture linear change:



$$\mathbf{y}_i = \mathbf{\Lambda}\boldsymbol{\eta}_i + \boldsymbol{\varepsilon}_i$$

$$\begin{bmatrix} y_{1i} \\ y_{2i} \\ y_{3i} \\ y_{4i} \\ y_{5i} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} \eta_{1i} \\ \eta_{2i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \\ \varepsilon_{5i} \end{bmatrix}$$

$$\boldsymbol{\eta}_i = \boldsymbol{\alpha} + \boldsymbol{\zeta}_i$$

$$\begin{bmatrix} \eta_{1i} \\ \eta_{2i} \end{bmatrix} = \begin{bmatrix} \alpha_1 + \zeta_{1i} \\ \alpha_2 + \zeta_{2i} \end{bmatrix}$$

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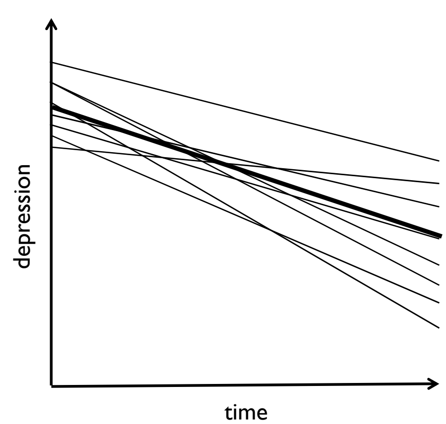
Linear LCM: Trajectories

- Intercept and linear slope model implies individual differences in both level and rate of change



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Heteroscedastic Residuals

- Standard LCM assumes each time-specific repeated measure is defined by a unique residual variance
  - called heteroscedasticity
- A simplifying condition is to assume error variances are equal
  - called homoscedasticity

$$\mathbf{\Theta} = \begin{bmatrix} \theta_{11} & & & & \\ 0 & \theta_{22} & & & \\ 0 & 0 & \theta_{33} & & \\ 0 & 0 & 0 & \theta_{44} & \\ 0 & 0 & 0 & 0 & \theta_{55} \end{bmatrix}$$

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Homoscedastic Residuals

- Can impose equality constraint on residuals over time
- This is a testable hypothesis
  - homoscedasticity more parsimonious, but may not correspond to characteristics of the observed data
- Typically want to identify most parsimonious structure that does not significantly contribute to model misfit

$$\mathbf{\Theta} = \begin{bmatrix} \theta & & & & \\ 0 & \theta & & & \\ 0 & 0 & \theta & & \\ 0 & 0 & 0 & \theta & \\ 0 & 0 & 0 & 0 & \theta \end{bmatrix}$$

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Summary

- The latent curve model fits logically within the CFA
- Estimate a latent factor for each trajectory component
- Enter numerical measure of time via factor loading matrix
- Can build basic to complex models and test intervening steps
- Can compare homo- vs. heteroscedasticity
- Can expand to nonlinear trajectories (not shown here)

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Section 2

Demonstration: Trajectories of Antisocial Behavior

Example: Antisocial Behavior

- N=405 cases drawn from National Longitudinal Survey of Youth
  - same core data as used in ARCL model in Chapter 3
  - but here do not have to pair adjacent ages -- advantage of LCM
- Age 6-8 at fist assessment; assessed up to 3 more times every-other year
- Mother report of child antisocial behavior on six items, each with a 0,1,2 response scale. Sum score ranges from 0 to 12
- Initial research questions:
  - what is the optimal mean functional form of the trajectory over time?
  - is there individual child-to-child variability around these mean values?
- Later we will consider child-specific predictors of the trajectories

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Example: Antisocial Behavior

- Accelerated longitudinal cohort design where *age* is time metric
- Accelerated cohorts results in missingness introduced by design
  - "x" denotes data that are present ; "•" denotes data that are missing

cohort	6	7	8	9	10	11	12	13
1	x	•	x	•	x	•	x	•
2	•	x	•	x	•	x	•	x
3	•	•	x	•	x	•	x	•
N	122	168	146	192	151	174	135	173

  - design allows for an *eight year* age span even though no individual child was assessed more than *four times*
  - also remember total number of children is 405

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Motivating Questions

- Is there evidence of individual variability in the overall level antisocial behavior?
- Does the inclusion of a linear slope factor significantly improve model fit?
- Is there evidence of individual variability in starting point and rate of change over time?
- Does the model support the constraint that the time-specific residuals be homoscedastic?
- Does the final model adequately fit the observed data?

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Antisocial Behavior Age-Specific Means

- Note that although any individual case has between one and four repeated assessments, there are eight unique ages

Age	*N*	*Mean*	*Variance*
6	122	1.57	2.76
7	168	1.60	2.41
8	146	1.90	3.34
9	192	1.99	3.83
10	151	2.01	4.48
11	174	1.91	3.95
12	135	1.88	3.54
13	173	2.38	5.25

- Because of missing data, these statistics are *ML Estimates*

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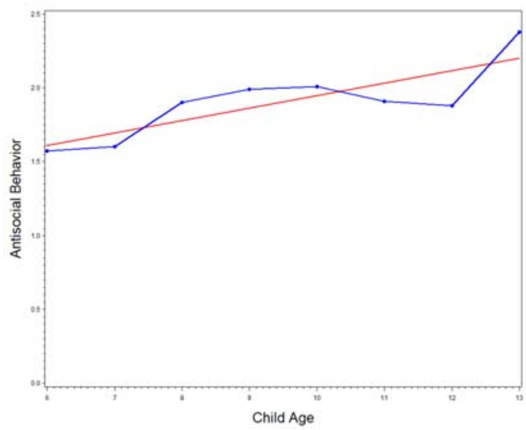
Antisocial Behavior Age-Specific Means

- Helpful to visually plot age-specific means



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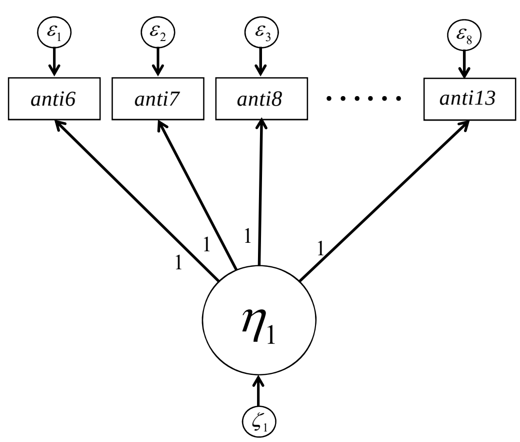
Intercept-Only Model

- We will begin by estimating an intercept-only model
  - although we expect this to fit poorly given the increasing sample means



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Intercept-Only Model

- As expected, this model fit the data quite poorly
  - $\chi^2(31) = 95.55$, $p < .0001$
  - CFI=.79
  - TLI=.83
  - RMSEA=.07
  - SRMR=.16
- No index supports adequate fit of the model to the sample data
- Next expand model to include a linear slope component
  - because intercept-only model is nested within the linear model, can conduct an LRT to assess improvement in model fit

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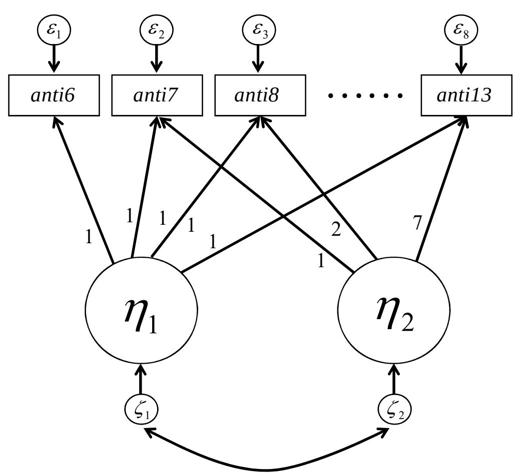
Intercept and Linear Slope Model

- We added a slope factor to the intercept-only model



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Intercept and Slope Model

- Fit indices reflects that linear model fits data rather well
  - $\chi^2(28)$ =42.88, p=.04;
  - CFI=.95; TLI=.96;
  - RMSEA=.036;
  - SRMR=.12
- Model fit data significantly better compared to intercept-only:
  - intercept-only: $\chi^2(31) = 95.55$
  - intercept + slope model: $\chi^2(28) = 42.88$
  - LRT difference: $\chi^2(3) = 52.67$ (p<.0001)
- Will retain slope given improvement in model fit

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Intercept and Slope Model

- Model defined to have heteroscedastic residuals
  - all age-specific residuals take on unique values
- Model could be simplified if single value estimated for all ages
  - prefer models with fewer parameters as they are *parsimonious*
- This is *homoscedasticity* and is a testable hypothesis
  - will conduct an LRT to compare heteroscedastic vs. homoscedastic

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Testing for Homoscedasticity

- Prior model defined *heteroscedastic* errors: diagonal elements differ

$$\mathbf{\Theta} = \begin{bmatrix} 1.74 & & & & & & & \\ 0 & 1.27 & & & & & & \\ 0 & 0 & 1.64 & & & & & \\ 0 & 0 & 0 & 1.85 & & & & \\ 0 & 0 & 0 & 0 & 2.16 & & & \\ 0 & 0 & 0 & 0 & 0 & 1.71 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.11 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.62 \end{bmatrix}$$

- Can impose *equality constraint* on residuals and conduct an LRT to evaluate potential decrement in model fit.

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Testing for Homoscedasticity

- Prior model defined heteroscedastic errors:

$$\chi^2(df = 28) = 42.88$$

- Model imposing equality constraints:

$$\chi^2(df = 35) = 50.07$$

- Resulting LRT is non-significant.

$$\chi^2_\Delta(df = 7) = 7.19, p = .66$$

- Supports retaining the homoscedasticity as most parsimonious model.

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for Homoscedasticity Testing

- Heteroscedastic :

$$\mathbf{\Theta} = \begin{bmatrix} 1.74 & & & & & & & \\ 0 & 1.27 & & & & & & \\ 0 & 0 & 1.64 & & & & & \\ 0 & 0 & 0 & 1.85 & & & & \\ 0 & 0 & 0 & 0 & 2.16 & & & \\ 0 & 0 & 0 & 0 & 0 & 1.71 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.11 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.62 \end{bmatrix}$$

- Homoscedastic:

$$\mathbf{\Theta} = \begin{bmatrix} 1.71 & & & & & & & \\ 0 & 1.71 & & & & & & \\ 0 & 0 & 1.71 & & & & & \\ 0 & 0 & 0 & 1.71 & & & & \\ 0 & 0 & 0 & 0 & 1.71 & & & \\ 0 & 0 & 0 & 0 & 0 & 1.71 & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.71 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.71 \end{bmatrix}$$

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Model Results

- The final model fit the data quite well

$$\chi^2(df = 35) = 50.07,\ p = .048$$
$$CFI = .95$$
$$TLI = .97$$
$$RSMEA = .033$$
$$SRMR = .12$$

- It is often easiest to see the final parameter estimates as part of a path diagram

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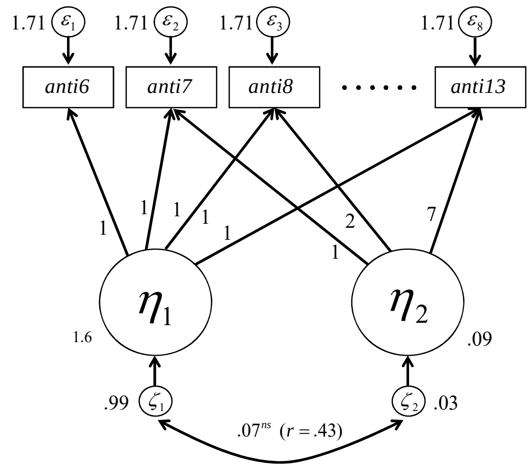
Final LCM Estimates

- All parameters are significant except for factor covariance



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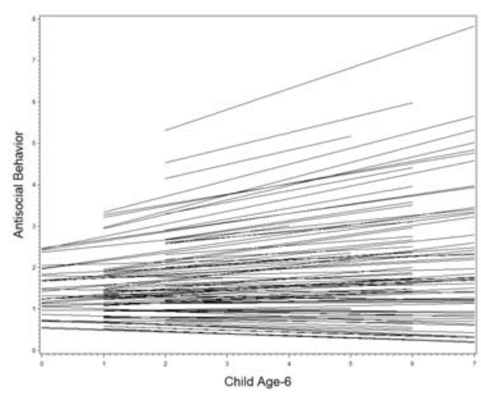
Estimated Individual Trajectories

- We can obtain estimates of first 100 individual trajectories



- clearly much variability in starting point and rate of change

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Summary

- Fitting a series of nested latent curve models showed linear model with homoscedastic residuals fit well
- The mean trajectory indicated that, on average, adolescents reported significant increases in antisocial behavior over time
- There was significant individual variability in both the intercept and slope indicating that some subjects reported higher vs. lower initial values and increased at slower vs. faster rates over time
- Final unconditional model suggests that one or more predictors of growth could provide insight into what type of child starts higher and increases more slowly over time

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Section 3

Time-Invariant Covariates

Time-Invariant Predictors (TICs)

- TICs are person-specific characteristics where value is constant over time
- Examples include biological sex, country of origin, DNA
  - could assess TICs at any time during study and would take on same value
- Can sometimes consider a one-time assessment of a construct as a TIC even though it could potentially change over time
  - e.g., marital status, family income, number of children, etc.
  - care must be taken here because could draw erroneous conclusions about influence of TIC if construct truly changing over time
- TICs incorporated in LCM as predictors of latent growth factors
  - shift in values of TICs shift conditional means of latent factors

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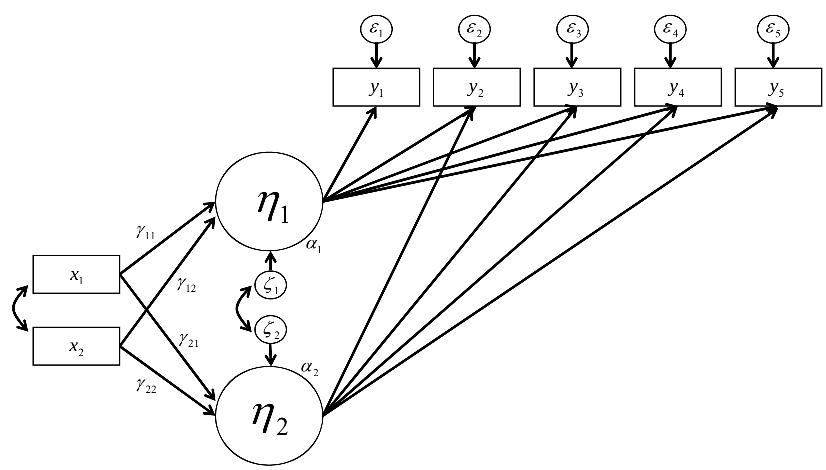
Time-Invariant Predictors



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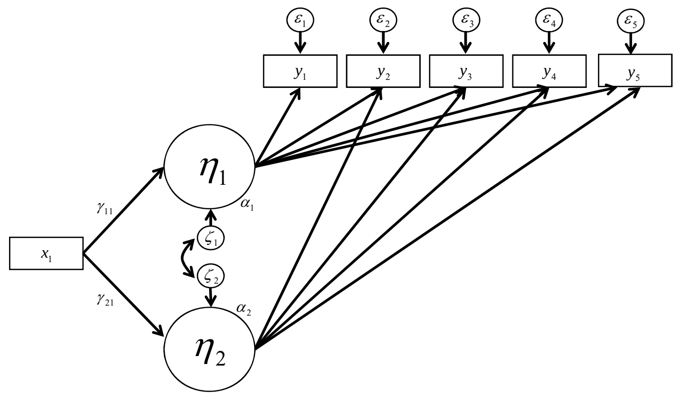
Probing Conditional Effects: 1 Predictor

- Regression of intercept on the TIC reflects a γ_{11}-unit shift in mean of intercept per one-unit shift in the TIC
- Regression of slope on the TIC reflects a γ_{21}-unit shift in mean of slope per one-unit shift in the TIC
- Can probe this effect as a function of time



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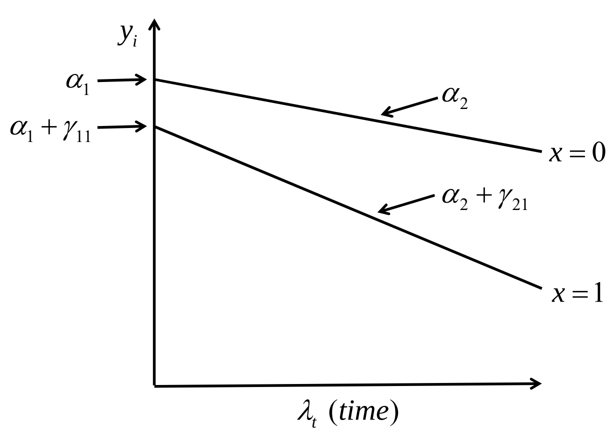
Probing Conditional Effects

- There are two unique model-implied trajectories, each defined by a *simple-intercept* and a *simple slope* for values 0 and 1 on the TIC



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Probing Conditional Effects

- Straightforward to probe conditional effect for a binary predictor because there are only two possible values: 0 and 1
- More challenging to probe effect of continuous predictor because theoretically an *infinite* number of values to consider
- Must choose a small number of *candidate values* of TIC at which to probe the effect
  - called a *pick-a-point* approach to probing interactions
- Many strategies for picking candidate points are defensible
  - e.g., clinical cut-offs, inter-quartile range
- Here we consider a standard deviation above/below the mean of the TIC

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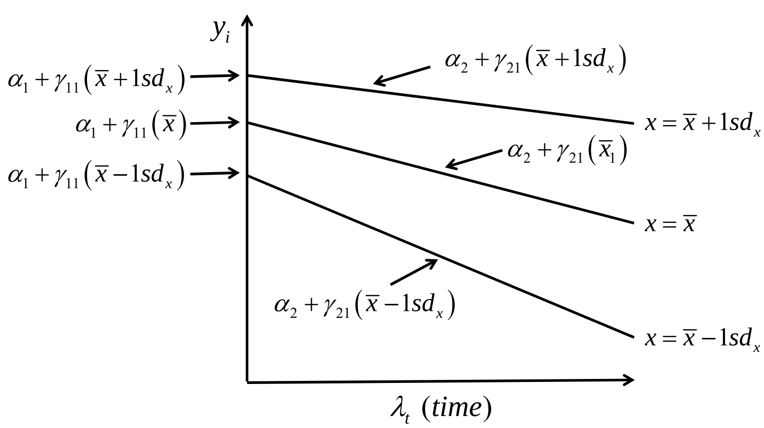
Probing Conditional Effects



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Trajectories of Antisocial Behavior

- Our best fitting model for antisocial behavior was a linear latent curve model with homoscedastic residuals
  - $\chi^2(df = 35) = 50.07,\ p = .048,\ RMSEA = .03, CFI = .95, TLI = .97$
  - significant mean intercept (1.6) and slope $(.09)$
  - significant variability in intercept and slope
  - non-significant covariance between intercept and slope
- Implies child-to-child variability in developmental trajectories of antisocial behavior between ages 6 and 13
- Suggests model could support one or more predictors of growth

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Trajectories of Antisocial Behavior

- We consider three time-invariant covariates
  1. Binary measure of biological sex:
     - MALE: 0=girl, 1=boy
  2. Mean-centered continuous measure of support of child's cognitive development in the home
     - HOMECOG: higher levels reflect greater support
  3. Mean-centered continuous measure of support of child's emotional development in the home
     - HOMEEMO: higher levels reflect greater support

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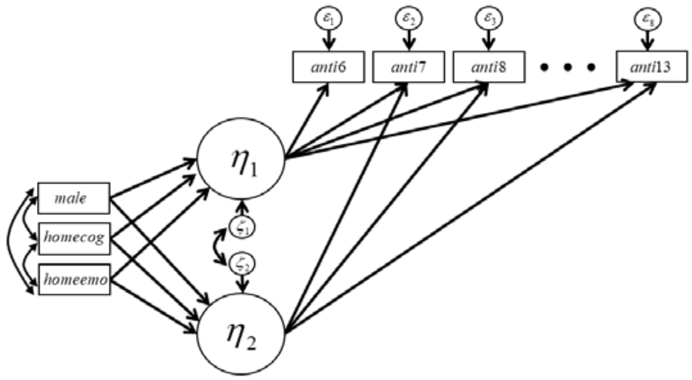
Conditional Linear LCM

- The path diagram for our proposed model is:



- The model fits quite well: $\chi^2(df = 53) = 77.88,\ p = .015,\ RMSEA = .034$

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Conditional Linear LCM

Predictor	Intercept	Slope
Intercept	1.193, p<.001	.078, p=.005
MALE	.817, p<.001	.015, p=.69
HOMECOG	−.025, p=.45	−.017, p=.028
HOMEEMO	−.17, p<.001	.008, p=.41
Multiple R^2	.34	.07

- Females and children from a home with higher levels of emotional support report *lower initial levels* of antisocial behavior
- Children from a home with higher levels of cognitive support report *less steep* increases in antisocial behavior over time
- We can probe these effects to better understand the relations

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Probing The Effect of Biological Sex

- First compute model-implied trajectories for boys and girls

Sex	Intercept	Slope
Female	1.193, p<.001	.078, p<.001
Male	2.01, p<.001	.094, p<.001

- Boys are starting at higher initial levels of antisocial behavior but both groups are increasing at a similar rates over time
  - know it is higher initial levels for boys because sex significantly predicted intercept factor
  - know it is similar rates of change for boys and girls because sex did not significantly predict slope
- It is easier to see these relations in a plot

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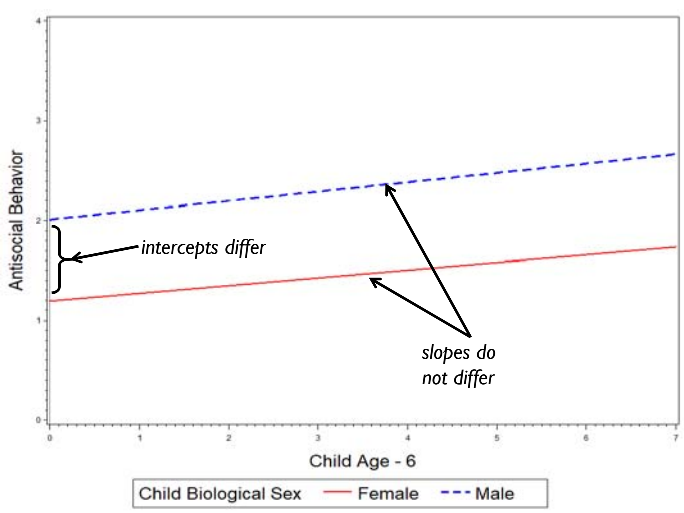
Probing The Effect of Biological Sex



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Probing The Effect of HOMECOG

- Will probe effects of HOMECOG because significant predictor of slope
  - must compute conditional trajectories to understand nature of effect
- Probing a binary predictor is straightforward: only two possible values
- Probing a continuous predictor is more challenging
  - in principle, there are an infinite number of possible conditional values
- Many strategies available for selecting conditional values
- Select one *sd* below mean, at the mean, and one *sd* above mean
  - because mean centered, the sample mean equals zero
  - sample standard deviation equals 2.57

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Probing The Effect of HOMECOG

- Simple slopes at low, medium and high cognitive support are:

HOMECOG	Intercept	Slope
Below the Mean	1.257, p<.001	.123, p<.001
At the Mean	1.193, p<.001	.078, p=.005
Above the Mean	1.129, p<.001	.033, p=.334

- Higher initial values of cognitive support are associated with systematically *lower initial levels* of antisocial behavior
- Higher initial values of cognitive support are associated with increasingly *less steep* slopes of antisocial behavior
  - indeed, slope of antisocial behavior is not significantly different from zero for children one *sd* above the mean of cognitive support

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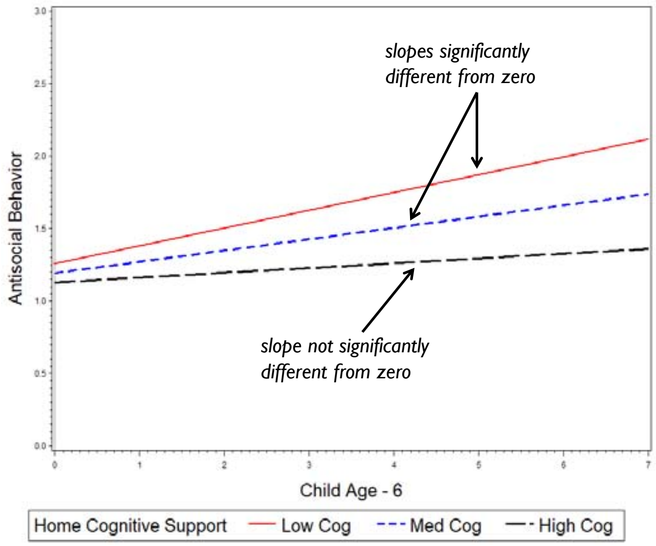
Probing The Effect of HOMECOG



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Summary

- Unconditional LCM showed significant variability in intercept and slope factors of antisocial behavior
- Conditional LCM showed the joint contributions of biological sex, cognitive support, and emotional support
- Girls and children from homes with higher emotional support report lower initial levels of antisocial behavior
- Children from homes with higher cognitive support report less steep increases in antisocial behavior over time
  - trajectories not significantly different from zero at highest levels of cognitive support

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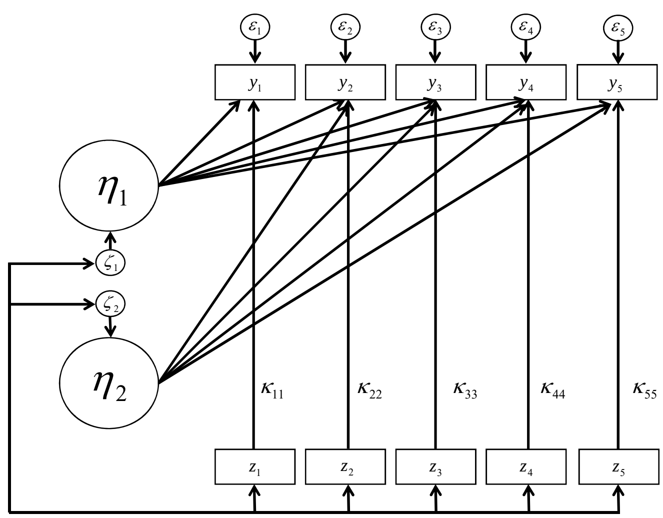
Section 4

Extensions of the LCM

Time-Varying Covariates: Unconditional



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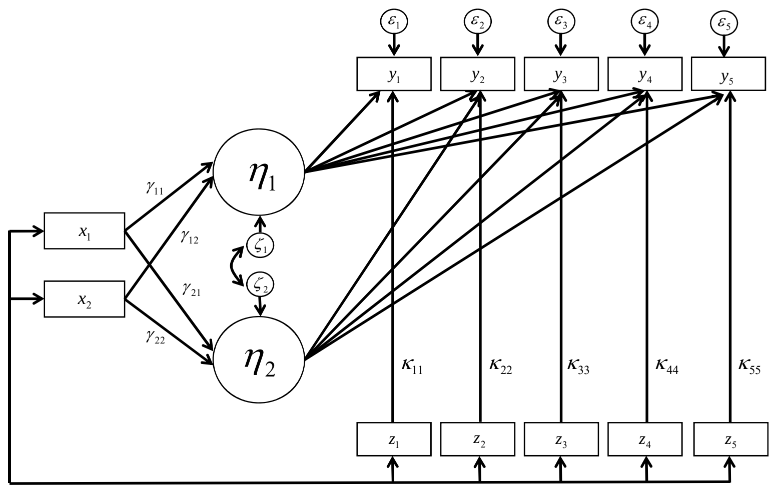
Conditional TVC Model



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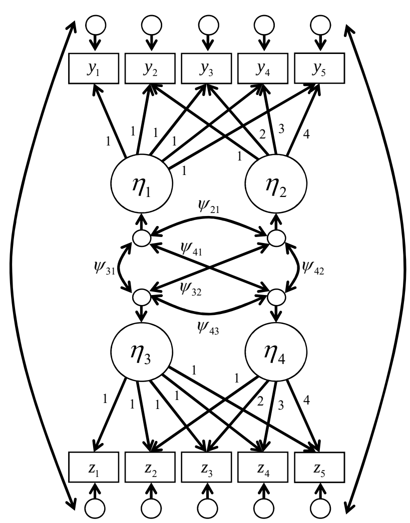
Multivariate LCM: Correlated Factors



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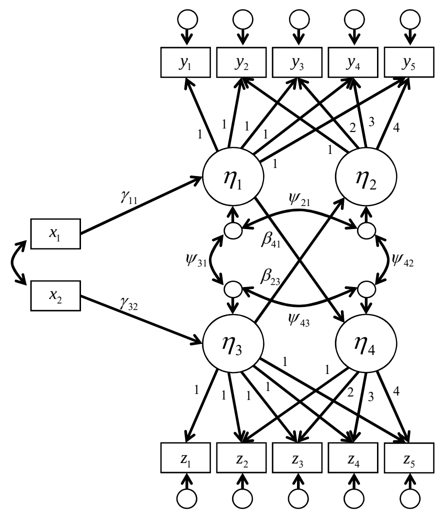
Multivariate LCM: Conditioned on TICs



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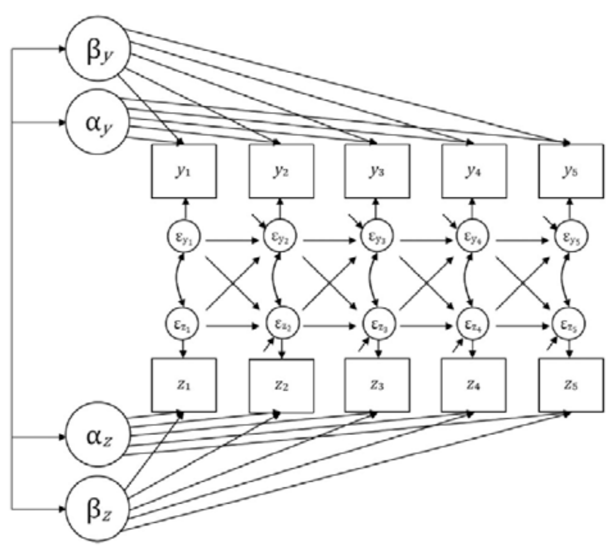
LCM with Structured Residuals



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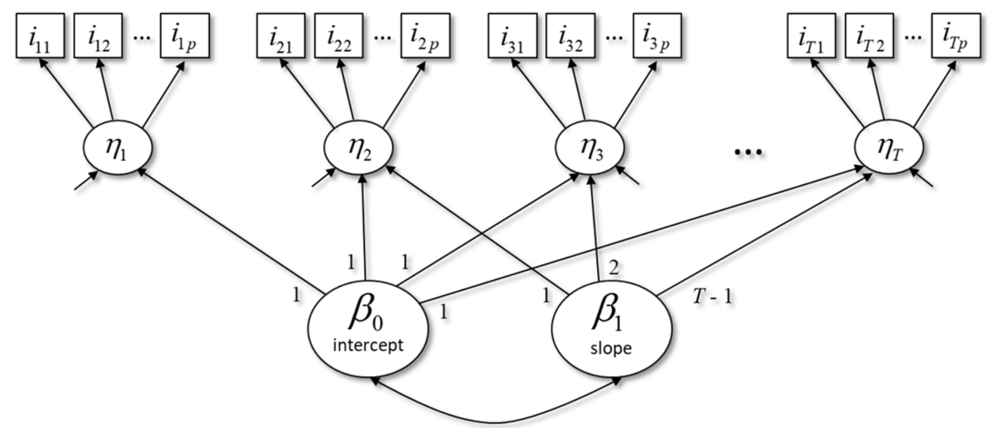
Second-Order Growth Model



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Summary

- Because LCM "lives" within the full structural equation modeling framework, many advanced models are possible
- These can be thought of as a "white board problem" where competing models are posited to relate the observed measures to one another
- Many additional advanced models have not been shown here: mixture modeling, growth factors as predictors, nonlinear trajectories, etc.
- The core challenge is to structure a given model to maximize the correspondence to theory

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Section 5

Comparison to the Multilevel Model

The Multilevel Growth Model

- The SEM (and general linear model) assumes independence
- In contrast, the multilevel model (or MLM) allows for nested (dependent) data
  - students nested within classroom, clients nested within therapist
- LCM motivated by repeated measures serving as indicators on latent factors
- In contrast, the multilevel-based growth model is motivated by the repeated measures being nested within individual
  - random effects play the same role as growth factors in LCM
- In many repeated measures designs where individuals are independent, the LCM and MLM are numerically identical across a broad range of applications
  - linear, nonlinear, conditional, TVCs, multiple group, etc.

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Relation of Multilevel Model to LCM

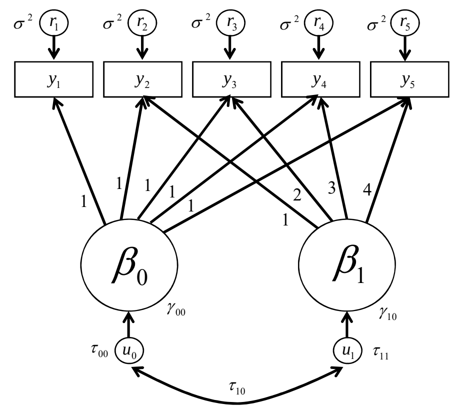
- Level-1 model

$$y_{ij} = \beta_{0j} + \beta_{1j} time_{ij} + r_{ij} \qquad r_{ij} \sim N(0, \sigma^2)$$

- Level-2 model

$$\beta_{0j} = \gamma_{00} + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + u_{1j}$$
$$\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \tau_{00} & \\ \tau_{10} & \tau_{11} \end{bmatrix} \right)$$

Same model, different notation and set up



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Compared to the LCM

- Despite equivalence for many models, there are important points where LCM and MLM diverge
- There are three core areas where the MLM tends to be advantageous

1. if the subjects themselves are nested in higher structures
   - repeated measures nested within client, and client nested within therapist
2. if there are a large number of repeated measures
   - "intensive" longitudinal data, e.g., 20+ per-person as part of a diary study
3. if the assessment schedule is individual-specific
   - Randomly-timed experience sampling where few or even no people share the same time assessment

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Compared to the SEM

- In contrast, the SEM-based LCM tends to be well suited for:
  1. Multivariate growth processes, especially with reciprocal effects
  2. Multiple-indicator latent factors to control for measurement error
  3. Formal tests of multi-chain mediation
  4. Advanced applications such as moderated nonlinear factor analysis
- If higher level nesting is present but a nuisance (i.e., no multilevel research questions) cluster-corrected standard errors can be used to obtain valid p-values / CIs despite non-independence
- Both approaches are powerful, the best of which depends on the data and research question at hand

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Summary

- For many growth modeling applications, LCM and MLM are identical and will return the same estimates
  - Different way of structuring data and fitting model, but ultimately equivalent
  - Same estimates will be obtained when same structure to model and same estimator used
- MLM and LCM offer contrasting advantages
  - MLM framework more readily accommodates complex nesting structures, many repeated measures, and individual-specific assessment schedules
  - LCM framework more readily allows for extended model specifications, such as second-order growth or multivariate models

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Workshop Summary

- Latent curve model is a powerful analytic approach for capturing individual differences in within-person change
- Often first summarize average trajectory and variance around this, then transition to predicting differences in individual trajectories
- Many extensions that permit other questions to be addressed
  - Time-varying covariates, multivariate models, reciprocal effects, second-order growth models, etc.
- Clear relationship between LCM and MLM approaches for modeling growth
  - Equivalent for basic models but different extensions to more complex models

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